

Young Researchers Seminar 2009

Torino, Italy, June 3-5, 2009

Short-time estimation of the OD matrix at a signalised junction by multisensor data fusion

Krystyna Biletska

INRETS/GRETIA
UTC/Heudiasyc



Contents

- 1 Introduction
- 2 OD matrix estimation problem
 - Problem statement
 - Data description
 - Methodology
- 3 Traffic flow modelling using high-level Petri nets
 - Fundamentals of HLPN
 - Application of HLPN to traffic flow modelling
- 4 Fuzzy linear programming for OD matrix estimation
 - Fuzzy modelling of the data
 - Fuzzy linear programming approach
 - Experimental results
- 5 Conclusion

Introduction

OD matrix at a junction

Knowledge of a traffic demand at a junction is represented by an **origin-destination** (OD) matrix.

OD Matrix (Definition)

$$\mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nm} \end{pmatrix}$$

where b_{ij} is an **OD flow rate**, the proportion of the flow of vehicles that come from entrance i during some period of time and go to exit j

Introduction

OD matrix at a junction

Knowledge of a traffic demand at a junction is represented by an **origin-destination (OD) matrix**.

OD Matrix (Definition)

$$\mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nm} \end{pmatrix}$$

where b_{ij} is an **OD flow rate**, the proportion of the flow of vehicles that come from entrance i during some period of time and go to exit j

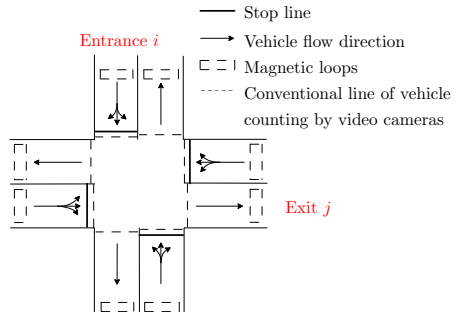


FIG.: Example of junction structure

$\mathbf{B}$ is deduced from **vehicle counts** made on each income and outcome link of the junction during a given time interval.

Introduction

OD matrix for a junction in transport systems

OD matrix estimated for a junction is a fundamental component for most of **transport systems**. It is used for :

- **traffic planning** purposes

Introduction

OD matrix for a junction in transport systems

OD matrix estimated for a junction is a fundamental component for most of **transport systems**. It is used for :

- **traffic planning** purposes
- estimation of **traffic demand** at a whole road network

Introduction

OD matrix for a junction in transport systems

OD matrix estimated for a junction is a fundamental component for most of **transport systems**. It is used for :

- **traffic planning** purposes
- estimation of **traffic demand** at a whole road network
- traffic **control strategies**

Introduction

OD matrix for a junction in transport systems

OD matrix estimated for a junction is a fundamental component for most of **transport systems**. It is used for :

- **traffic planning** purposes
- estimation of **traffic demand** at a whole road network
- traffic **control strategies**
- traveller information systems and validation of different traffic models

Introduction

OD matrix for a junction in transport systems

OD matrix estimated for a junction is a fundamental component for most of **transport systems**. It is used for :

- **traffic planning** purposes
- estimation of **traffic demand** at a whole road network
- traffic **control strategies**
- traveller information systems and validation of different traffic models
- diagnostic system for signalised junctions, developed at INRETS/GRETIA (Midenet *et al.*, 2004) which compares the impacts of different traffic control strategies formulated, for instance, in terms of **CO₂** and pollutant emissions.

Introduction

OD matrix for a junction in transport systems

OD matrix estimated for a junction is a fundamental component for most of **transport systems**. It is used for :

- **traffic planning** purposes
- estimation of **traffic demand** at a whole road network
- traffic **control strategies**
- traveller information systems and validation of different traffic models
- diagnostic system for signalised junctions, developed at INRETS/GRETIA (Midenet *et al.*, 2004) which compares the impacts of different traffic control strategies formulated, for instance, in terms of **CO₂** and pollutant emissions.

NB : Because of **temporal variability** of the traffic demand, most of these systems require **real-time knowledge** about OD matrix.

Introduction

Research objective

- Consider a signalised junction with n entrances, m exits and q inner zones

Introduction

Research objective

- Consider a signalised junction with n entrances, m exits and q inner zones
- Entrances and/or inner zones are controlled with traffic lights

Introduction

Research objective

- Consider a signalised junction with n entrances, m exits and q inner zones
- Entrances and/or inner zones are controlled with traffic lights
- Junction is fitted out with video cameras that provide every second :
 - vehicle counts $\mathcal{X}_i(\tau)$ at entrance i ($i = 1, \dots, n$)
 - vehicle counts $\mathcal{Y}_j(\tau)$ at exit j ($j = 1, \dots, m$)
 - $\mathcal{Z}_k(\tau)$ number of stopped vehicles at inner zone k ($k = 1, \dots, q$)

Introduction

Research objective

- Consider a signalised junction with n entrances, m exits and q inner zones
- Entrances and/or inner zones are controlled with traffic lights
- Junction is fitted out with video cameras that provide every second :
 - vehicle counts $\mathcal{X}_i(\tau)$ at entrance i ($i = 1, \dots, n$)
 - vehicle counts $\mathcal{Y}_j(\tau)$ at exit j ($j = 1, \dots, m$)
 - $\mathcal{Z}_k(\tau)$ number of stopped vehicles at inner zone k ($k = 1, \dots, q$)
- Estimate an OD matrix $\mathbf{B}$ at each traffic light cycle (duration of green-amber-red sequence)

Problem statement

For a given traffic light cycle c , let

- $x_i(c)$ be the flow volume at entrance i during c ($\forall i = 1, \dots, n$)
- $y_j(c)$ be the flow volume entered the junction during c and left it by exit j ($\forall j = 1, \dots, m$)
- $z_{kij}(c)$ be the number of vehicles which cross the junction from i to j and stop at inner zone k during c ($\forall k = 1, \dots, q$)
- b_{ij} be the OD flow rate

Problem statement

For a given traffic light cycle c , let

- $x_i(c)$ be the flow volume at entrance i during c ($\forall i = 1, \dots, n$)
- $y_j(c)$ be the flow volume entered the junction during c and left it by exit j ($\forall j = 1, \dots, m$)
- $z_{kij}(c)$ be the number of vehicles which cross the junction from i to j and stop at inner zone k during c ($\forall k = 1, \dots, q$)
- b_{ij} be the OD flow rate

The problem is to **estimate OD flow rates** b_{ij} at the end of each traffic light cycle c , such that

$$\begin{aligned}
 y_j(c) &= \sum_{i=1}^n b_{ij}(c) x_i(c), \\
 x_i(c) b_{ij}(c) &\geq z_{kij}(c) \quad \forall k \text{ s.t. } \delta_{kij} = 1, \\
 \sum_{j=1}^m b_{ij}(c) &= 1, \\
 b_{ij}(c) &\geq 0,
 \end{aligned}$$

where $\delta_{kij} = 1$ if OD flow from i to j can pass through inner zone k and is 0 otherwise.

Data description

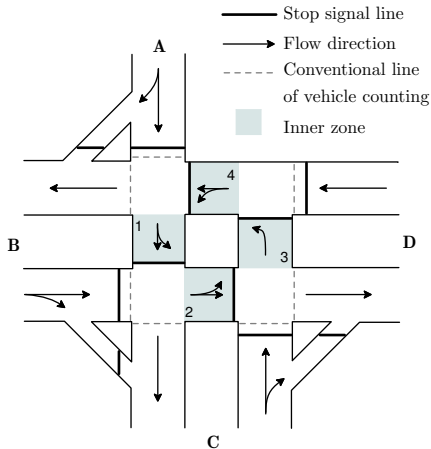


FIG.: The experimental site

Data description

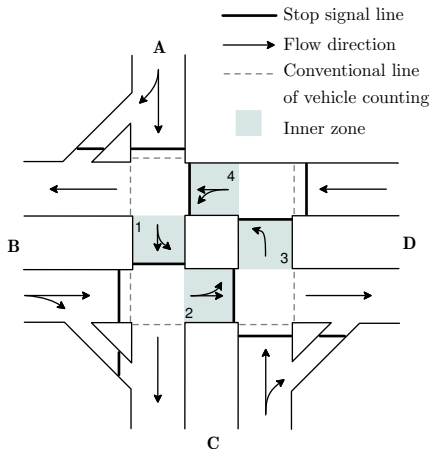


FIG.: The experimental site

Traffic measurements

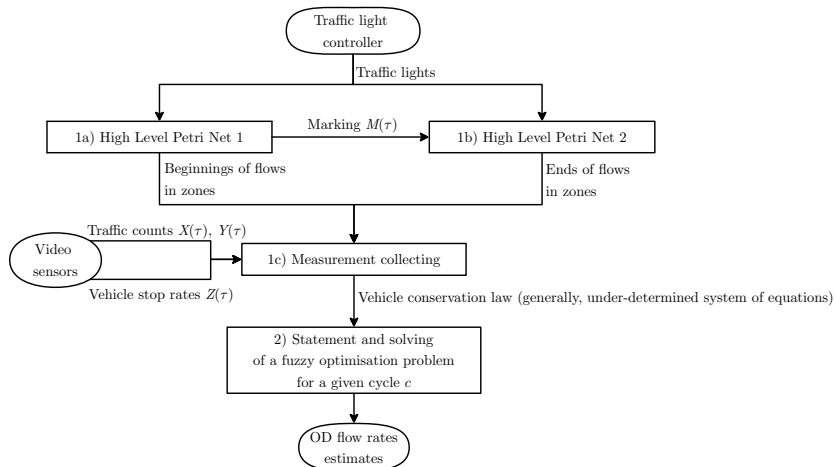
$$\mathcal{X}_i(\tau), \mathcal{Y}_j(\tau) \text{ and } \mathcal{Z}_k(\tau)$$

are available **every second τ**

The **quality measurement** of traffic parameters is influenced by

- traffic and meteorological conditions
- characteristics of vehicles

Methodology

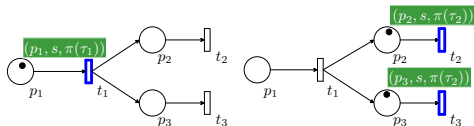
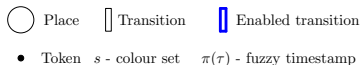


Fundamentals of HLPN

- A High-Level Petri Net (HLPN) is a graphical and mathematical tool used to model discrete-event systems (Murata 1989)

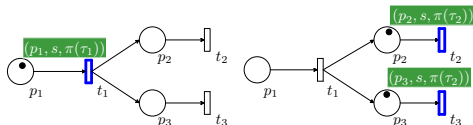
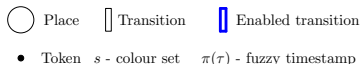
Fundamentals of HLPN

- A High-Level Petri Net (HLPN) is a graphical and mathematical tool used to model discrete-event systems (Murata 1989)
- In graphic representation, an HLPN is a directed graph composed of two kinds of nodes :
places p and **transitions** t



Fundamentals of HLPN

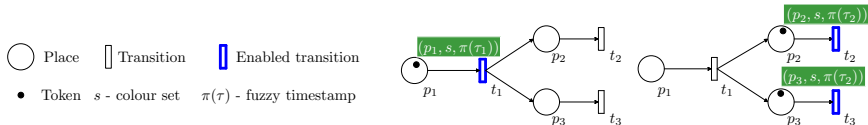
- A High-Level Petri Net (HLPN) is a graphical and mathematical tool used to model discrete-event systems (Murata 1989)
- In graphic representation, an HLPN is a directed graph composed of two kinds of nodes :
places p and **transitions** t



- A place may be **marked** by coloured **tokens** endowed with fuzzy timestamps and characterises the system state

Fundamentals of HLPN

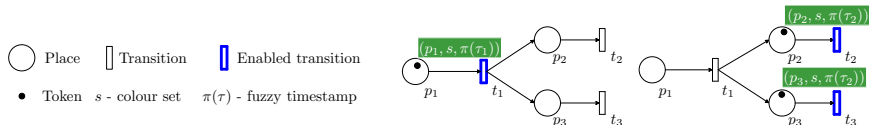
- A High-Level Petri Net (HLPN) is a graphical and mathematical tool used to model discrete-event systems (Murata 1989)
- In graphic representation, an HLPN is a directed graph composed of two kinds of nodes :
places p and **transitions** t



- A place may be **marked** by coloured **tokens** endowed with fuzzy timestamps and characterises the system state
- A transition represents an event that can change the system state

Fundamentals of HLPN

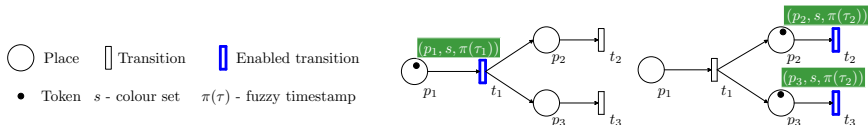
- A High-Level Petri Net (HLPN) is a graphical and mathematical tool used to model discrete-event systems (Murata 1989)
- In graphic representation, an HLPN is a directed graph composed of two kinds of nodes :
places p and **transitions** t



- A place may be **marked** by coloured **tokens** endowed with fuzzy timestamps and characterises the system state
- A transition represents an event that can change the system state
- When the event occurs at time τ the transition is **enabled** and the marking $\mathcal{M}(p, \tau)$ of input and output places changes

Fundamentals of HLPN

- A High-Level Petri Net (HLPN) is a graphical and mathematical tool used to model discrete-event systems (Murata 1989)
- In graphic representation, an HLPN is a directed graph composed of two kinds of nodes :
places p and **transitions** t



- A place may be **marked** by coloured **tokens** endowed with fuzzy timestamps and characterises the system state
- A transition represents an event that can change the system state
- When the event occurs at time τ the transition is **enabled** and the marking $\mathcal{M}(p, \tau)$ of input and output places changes
- Dynamic behaviour of the system is expressed by means of time-varying marking
 A formal representation of HLPN is described in (Jensen *et al.*, 1998 ; Biletska *et al.*, 2009)

Application of HLPN to traffic flow modelling

Structure

- Place stands for a zone of the junction

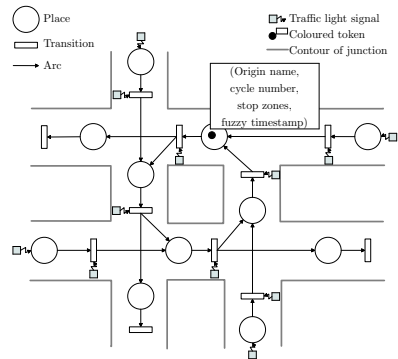


FIG.: High Level Petri Net 1

Application of HLPN to traffic flow modelling

Structure

- Place stands for a zone of the junction
- Transitions connected to enter and inner places represent traffic lights

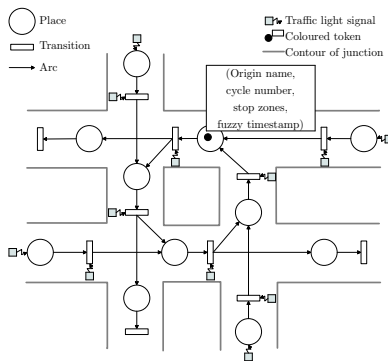


FIG.: High Level Petri Net 1

Application of HLPN to traffic flow modelling

Structure

- Place stands for a zone of the junction
- Transitions connected to enter and inner places represent traffic lights
- Token in place p , represented by a triple

$$(p, s, \pi(\theta)),$$

means a possible presence of an OD flow at the corresponding junction zone :

- colour set** $s = (s_1, s_2, s_3)$ contains name of flow origin $s_1 \in \{A, B, C, D\}$, cycle number $s_2 \in \mathbb{N}$ and set of inner zones s_3
- fuzzy timestamp** $\pi(\theta) \in [0, 1]$ is the degree of possibility that the token arrives in p at time θ

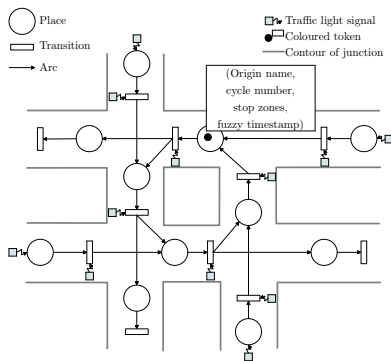
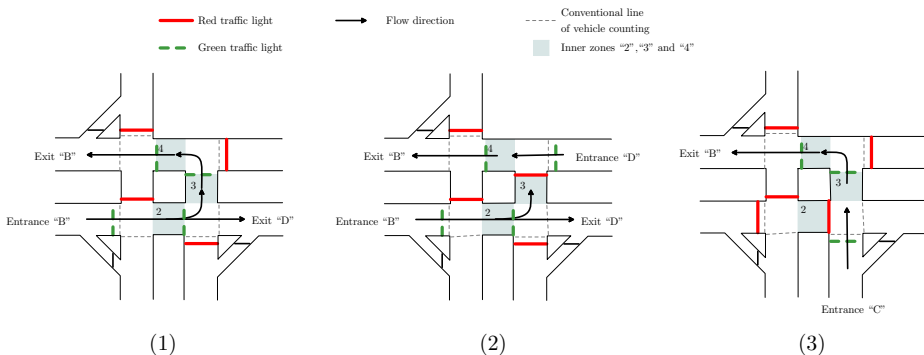


FIG.: High Level Petri Net 1

Application of HLPN to traffic flow modelling

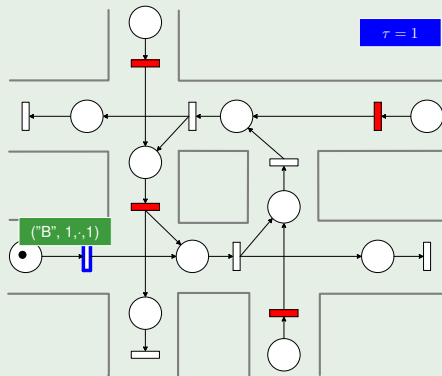
HLPN example. Scenario



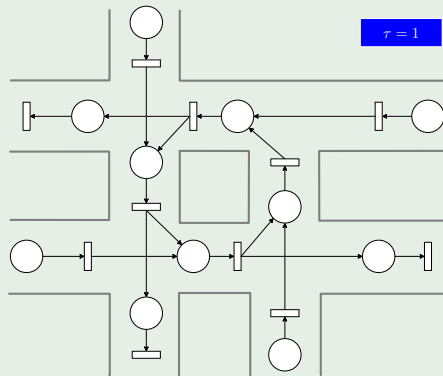
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



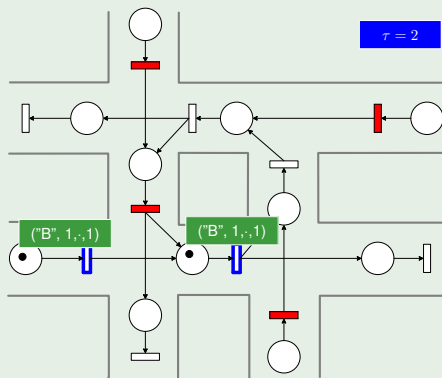
HLPN2



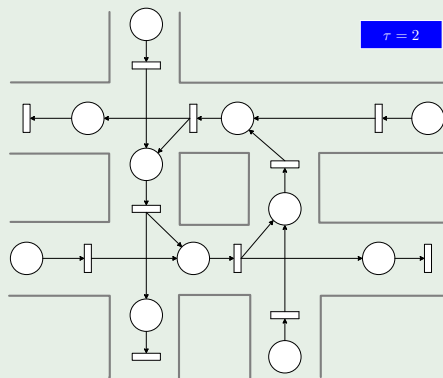
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



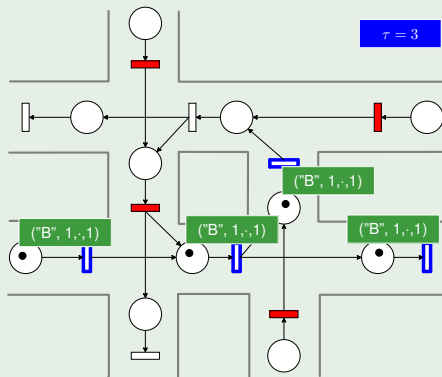
HLPN2



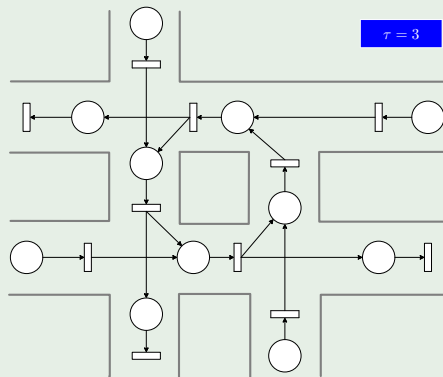
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



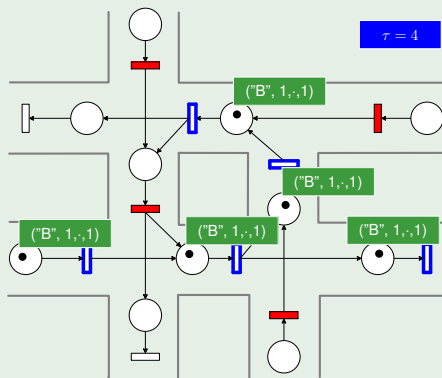
HLPN2



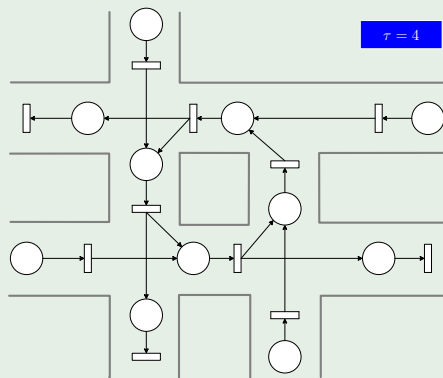
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



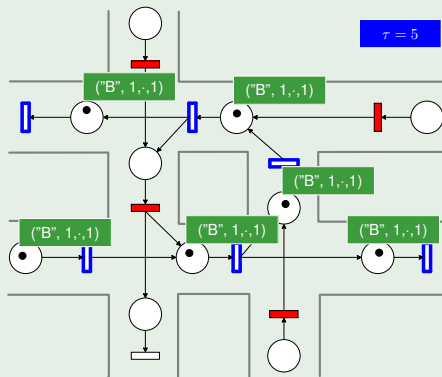
HLPN2



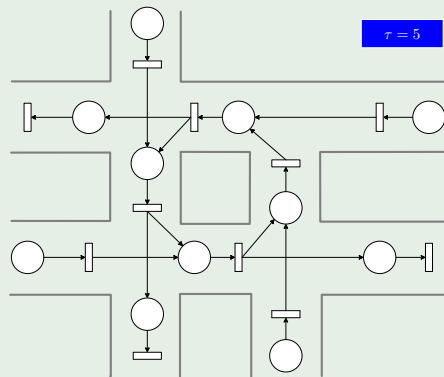
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



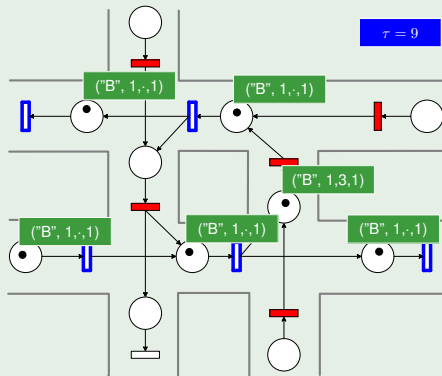
HLPN2



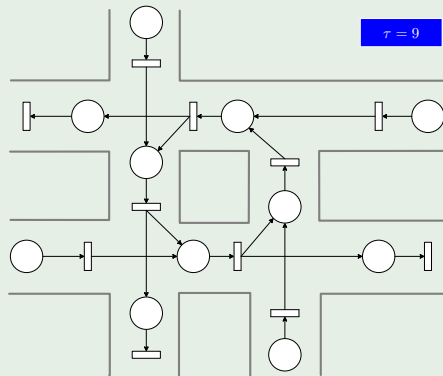
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



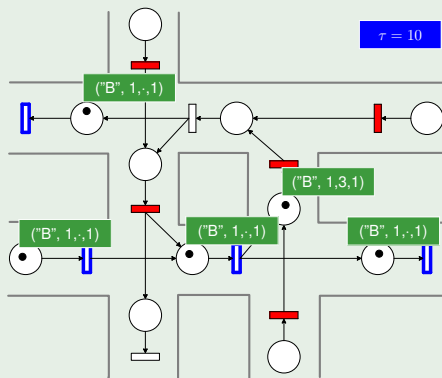
HLPN2



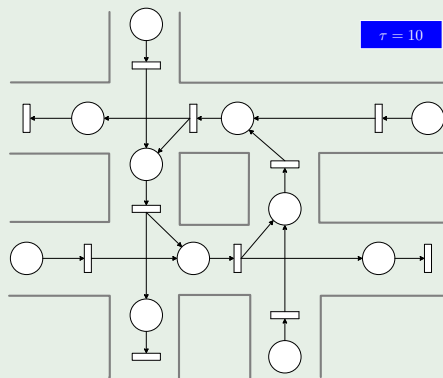
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



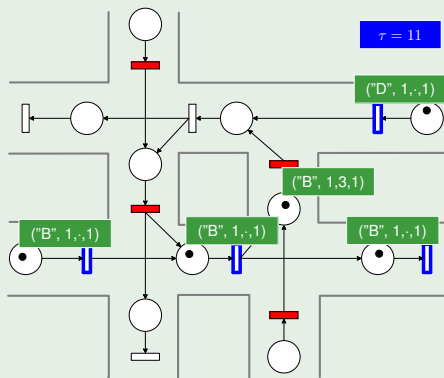
HLPN2



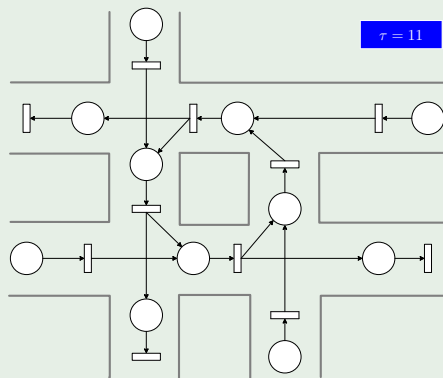
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



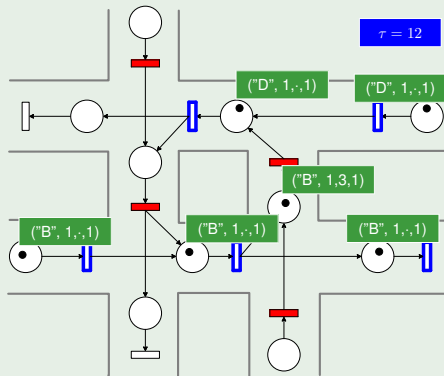
HLPN2



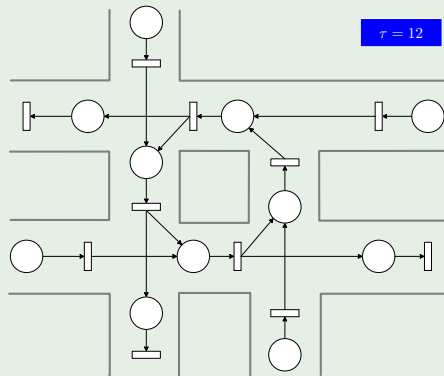
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



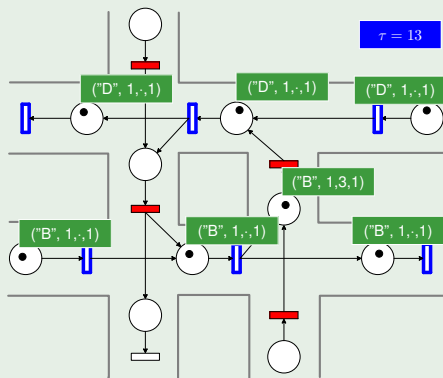
HLPN2



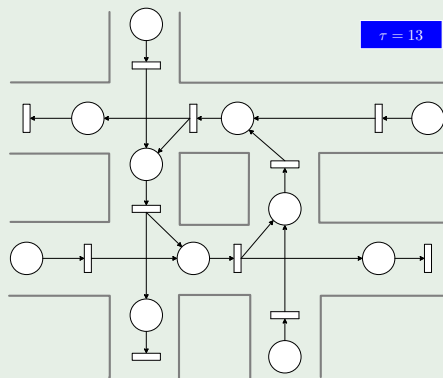
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



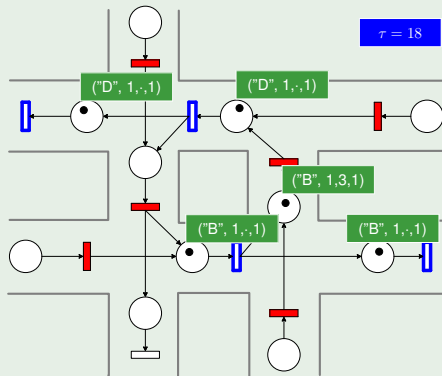
HLPN2



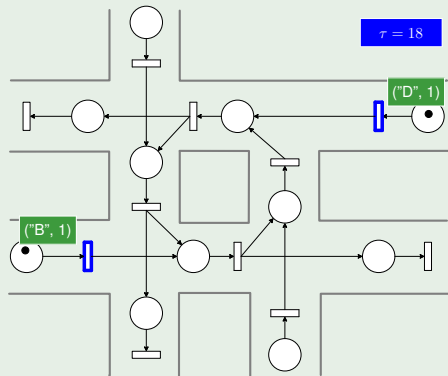
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



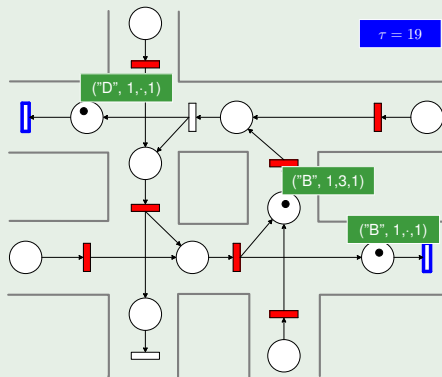
HLPN2



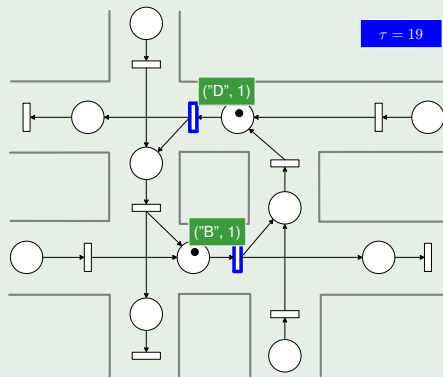
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



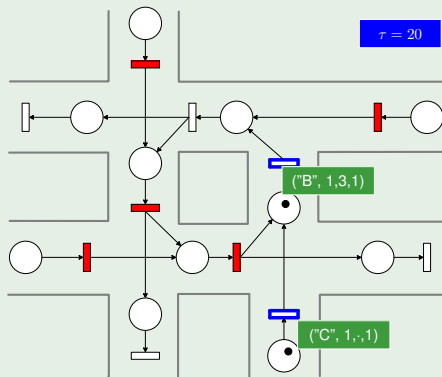
HLPN2



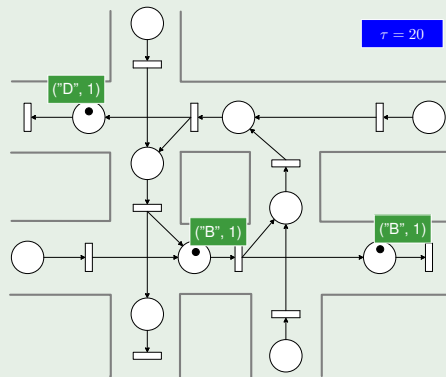
Application of HLPN to traffic flow modelling

HLPN example. Simulation

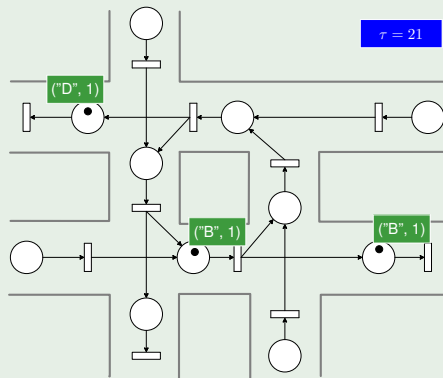
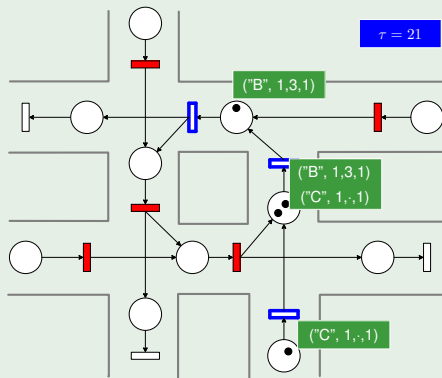
HLPN1



HLPN2



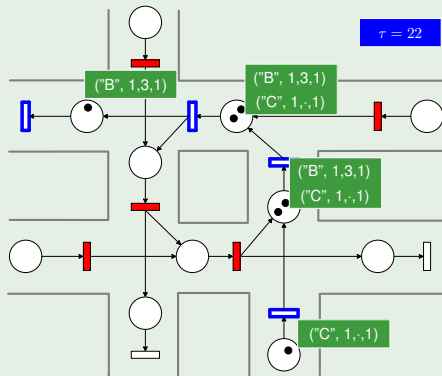
HLPN example. Simulation



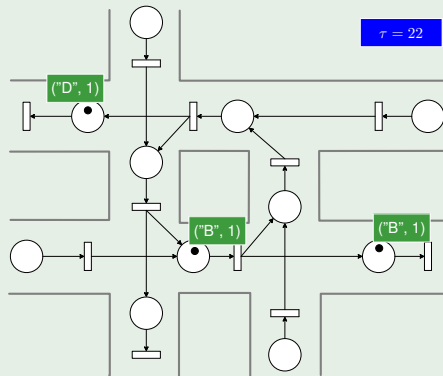
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



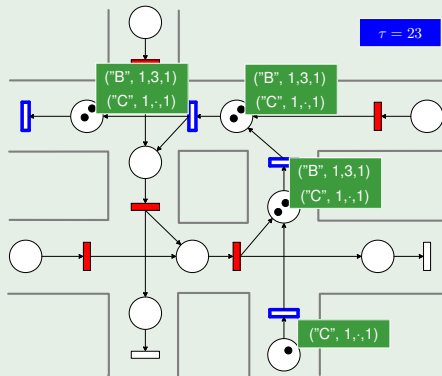
HLPN2



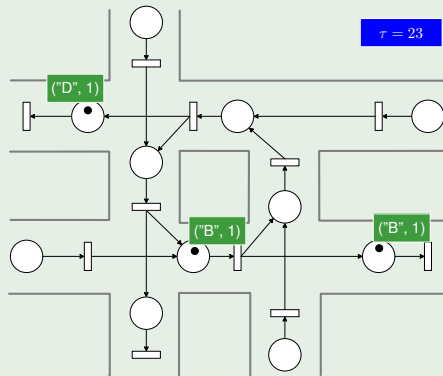
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



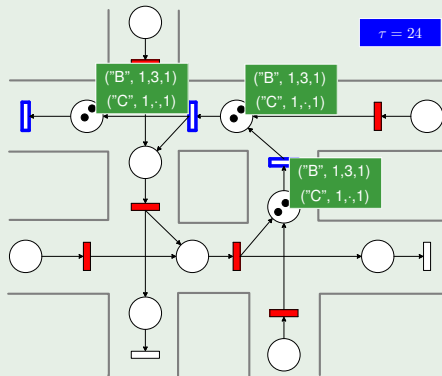
HLPN2



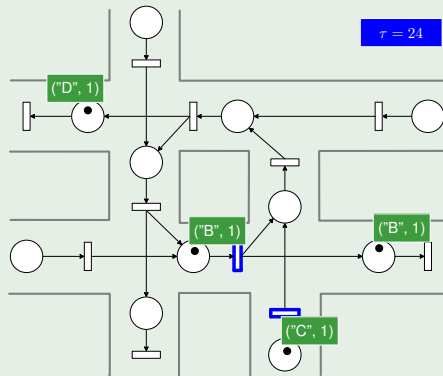
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



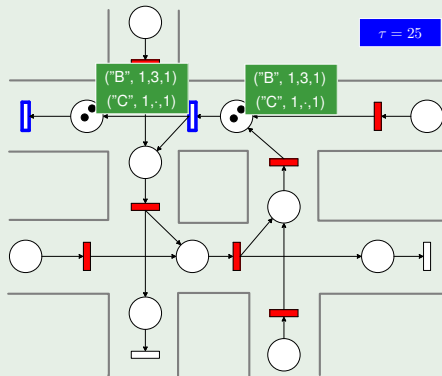
HLPN2



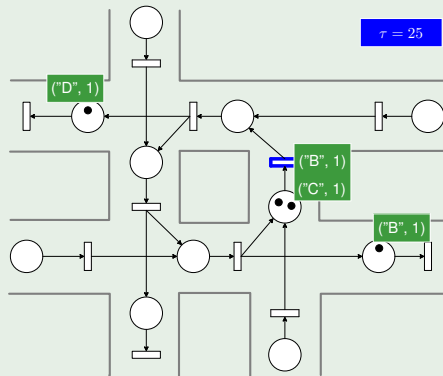
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



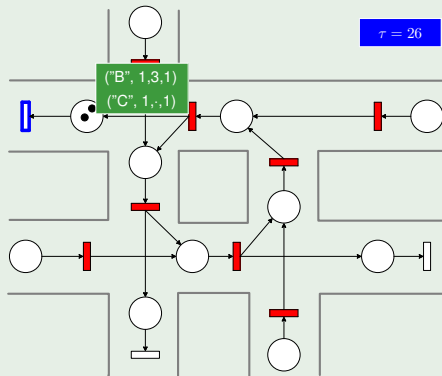
HLPN2



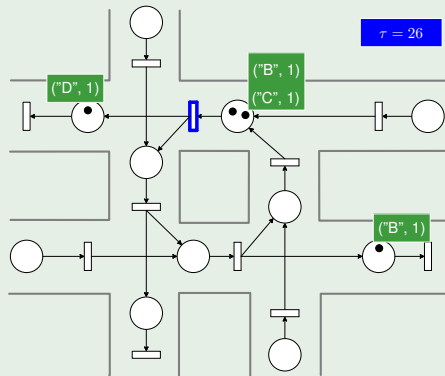
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



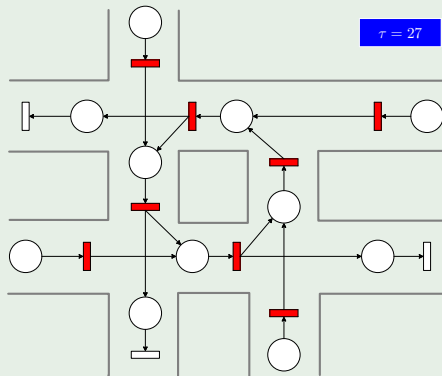
HLPN2



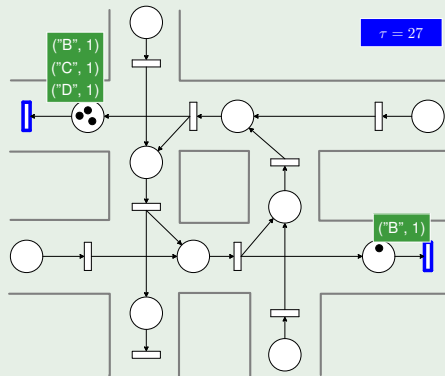
Application of HLPN to traffic flow modelling

HLPN example. Simulation

HLPN1



HLPN2



Application of HLPN to traffic flow modelling

Construction of vehicle conservation law

Estimate OD flow rates b_{ij} ($\forall i = 1, \dots, n$) at the end of each traffic light cycle c , such that

$$y_j(c) = \sum_{i=1}^n b_{ij}(c)x_i(c),$$

$$x_i(c)b_{ij}(c) \geq z_{kij}(c) \quad \forall k \text{ s.t. } \delta_{kij} = 1,$$

$$\sum_{j=1}^m b_{ij}(c) = 1,$$

$$b_{ij}(c) \geq 0,$$

where

- $x_i(c) = \sum_{\tau=1}^{\mathcal{G}_i(c)} \mathcal{X}_i(\tau)$ is an entrance flow volume
- $y_j(c)$ is segmented into $L_j(c)$ platoons $y_j^l(c) = \sum_{\tau \in \Delta_{\tau_l}(c)} \mathcal{Y}_j(\tau)$
- $z_{kij}^l(c) = \max_{\tau \in \mathcal{R}_k(c)} \mathcal{Z}_k(\tau) \delta_{kij}$

here $\mathcal{G}_i(c)$ and $\mathcal{R}_k(c)$ - green and red light duration of cycle c at i and k

Application of HLPN to traffic flow modelling

Construction of vehicle conservation law

Estimate OD flow rates b_{ij} ($\forall i = 1, \dots, n$) at the end of each traffic light cycle c , such that

$$y_j(c) = \sum_{i=1}^n b_{ij}(c)x_i(c),$$

$$x_i(c)b_{ij}(c) \geq z_{kij}(c) \quad \forall k \text{ s.t. } \delta_{kij} = 1,$$

$$\sum_{j=1}^m b_{ij}(c) = 1,$$

$$b_{ij}(c) \geq 0,$$

where

- $x_i(c) = \sum_{\tau=1}^{\mathcal{G}_i(c)} \mathcal{X}_i(\tau)$ is an entrance flow volume
- $y_j(c)$ is segmented into $L_j(c)$ platoons $y_j^l(c) = \sum_{\tau \in \Delta_{\tau_l}(c)} \mathcal{Y}_j(\tau)$
- $z_{kij}^l(c) = \max_{\tau \in \mathcal{R}_k(c)} \mathcal{Z}_k(\tau) \delta_{kij}$

here $\mathcal{G}_i(c)$ and $\mathcal{R}_k(c)$ - green and red light duration of cycle c at i and k

OD estimation problem (Matrix form)

Find $\mathbf{b}$ such that

$$\mathbf{y} = \mathbf{X}_1 \mathbf{b}$$

$$\mathbf{X}_2 \mathbf{b} \geq \mathbf{z}$$

$$\mathbf{1b} = \mathbf{1}$$

$$\mathbf{b} \geq 0$$

Fuzzy modelling of the data

Triangular fuzzy number

Triangular fuzzy number (Definition)

Triangular fuzzy number $\tilde{a}$ is represented by pair

$$(A, \mu_A(x)),$$

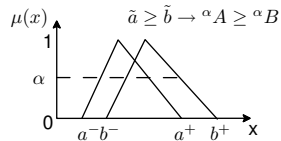
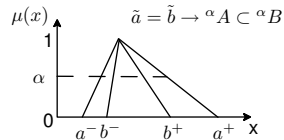
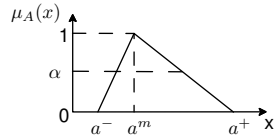
where $A = [a^-, a^+]$ is a **support** and

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{otherwise} \end{cases}$$

is a **membership function**. In this paper $A \subset \mathbb{R}^+$

When $\mu_A(x) = 1$, $x = a^m$ is said to be the **mode** (peak) of the fuzzy number

${}^\alpha A = \{x | \mu_A(x) \geq \alpha\}$ means the **α -cut** of the fuzzy number $\tilde{a}$



Fuzzy modelling of the data

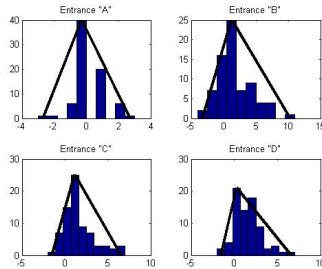
Real data modelling

- Let $\tilde{\mathbf{X}}_1$, $\tilde{\mathbf{X}}_2$, $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{z}}$ be triangular fuzzy numbers

Fuzzy modelling of the data

Real data modelling

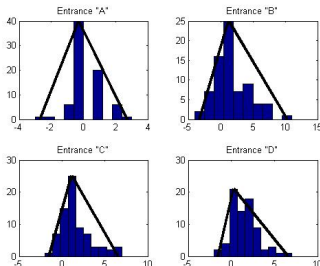
- Let $\tilde{X}_1, \tilde{X}_2, \tilde{y}$ and $\tilde{z}$ be triangular fuzzy numbers
- The form of fuzzy numbers $\tilde{X}_1$ is derived from empirical distributions of the error counts



Fuzzy modelling of the data

Real data modelling

- Let $\tilde{X}_1, \tilde{X}_2, \tilde{y}$ and $\tilde{z}$ be triangular fuzzy numbers
- The form of fuzzy numbers $\tilde{X}_1$ is derived from empirical distributions of the error counts

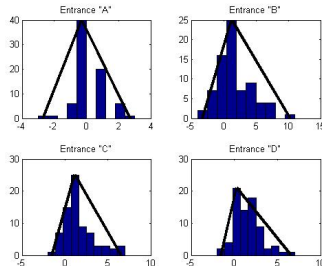


- Since there is no available histograms of error counts for y and z , the fuzzy numbers $\tilde{y}$ and $\tilde{z}$ are supposed to be symmetrical with the spreads chosen experimentally

Fuzzy modelling of the data

Real data modelling

- Let $\tilde{X}_1, \tilde{X}_2, \tilde{y}$ and $\tilde{z}$ be triangular fuzzy numbers
- The form of fuzzy numbers $\tilde{X}_1$ is derived from empirical distributions of the error counts



- Since there is no available histograms of error counts for y and z , the fuzzy numbers $\tilde{y}$ and $\tilde{z}$ are supposed to be symmetrical with the spreads chosen experimentally
- $X_1^-, X_2^-, y^-, z^- \in \mathbb{R}^+$ because of the nature of the data

Fuzzy linear programming approach

Supposing the unknown OD flow rates $\mathbf{b}$ to be triangular fuzzy numbers of the form $(\mathbf{b}^-, \mathbf{b}^m, \mathbf{b}^+)$, we write the following fuzzy linear programming problem :

$$\min_{\mathbf{b}^m, \mathbf{b}^+, \mathbf{b}^-, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3} \left(\sum_{\alpha=1}^N \sum_{i=1}^I (\alpha \mathbf{b}_i^+ - \alpha \mathbf{b}_i^-) + \gamma_1 \sum_j \mathbf{e}_{1,j} + \gamma_2 \sum_j \mathbf{e}_{2,j} + \gamma_3 \sum_k \mathbf{e}_{3,k} \right)$$

subject to

$$\left\{ \begin{array}{l} \alpha \mathbf{X}_1^- \mathbf{b}^- \leq \alpha \mathbf{y}^- + \mathbf{e}_1, \\ \alpha \mathbf{X}_1^+ \mathbf{b}^+ + \mathbf{e}_2 \geq \alpha \mathbf{y}^+, \\ \alpha \mathbf{X}_2^+ \mathbf{b}^+ + \mathbf{e}_3 \geq \alpha \mathbf{z}^-, \\ 0 \leq \alpha_i \mathbf{b}^- \leq \mathbf{b}^m \leq \alpha_i \mathbf{b}^+ \leq 1, \quad \forall i = 1, \dots, N \\ \alpha_i \mathbf{b}^- \leq \alpha_j \mathbf{b}^- \leq \alpha_j \mathbf{b}^+ \leq \alpha_i \mathbf{b}^+, \quad i < j, \forall i, j = 1, \dots, N, \\ \mathbf{lb}^m = \mathbb{1}, \\ \mathbf{e}_1 \geq 0, \mathbf{e}_2 \geq 0, \mathbf{e}_3 \geq 0, \end{array} \right.$$

Generalised form of constraints

$$\begin{aligned} \tilde{\mathbf{y}} &= \tilde{\mathbf{X}}_1 \tilde{\mathbf{b}} \\ \tilde{\mathbf{X}}_2 \tilde{\mathbf{b}} &\geq \tilde{\mathbf{z}} \\ 0 &\leq \tilde{\mathbf{b}} \leq 1 \\ \mathbf{lb}^m &= \mathbb{1} \\ \mathbf{e}_1 &\geq 0, \mathbf{e}_2 \geq 0, \mathbf{e}_3 \geq 0, \end{aligned}$$

Experimental results

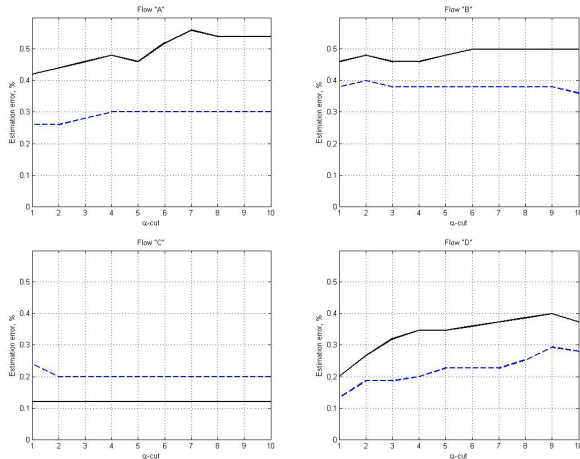


Fig.: Estimation error vs α -cuts for peak hour (black) and off-peak hour (blue) traffic conditions

Conclusion

- A new short-time estimation method of the OD matrix for a signalised junction is proposed. It proceeds in two steps :
 - An HLPN model provides a vehicle conservation law at each traffic light cycle
 - A fuzzy linear programming approach provides a point estimation endowed with a confidence interval

Conclusion

- A new short-time estimation method of the OD matrix for a signalised junction is proposed. It proceeds in two steps :
 - An HLPN model provides a vehicle conservation law at each traffic light cycle
 - A fuzzy linear programming approach provides a point estimation endowed with a confidence interval
- Data, used to estimate the OD flow rates, are collected at an experimental junction

Conclusion

- A new short-time estimation method of the OD matrix for a signalised junction is proposed. It proceeds in two steps :
 - An HLPN model provides a vehicle conservation law at each traffic light cycle
 - A fuzzy linear programming approach provides a point estimation endowed with a confidence interval
- Data, used to estimate the OD flow rates, are collected at an experimental junction
- The best results, obtained for $\alpha = 3$, are accurate and not significantly different from those obtained in our previous work (Biletska *et al.*). However, the confidence interval ensures a set of plausible solutions.

Conclusion

- A new short-time estimation method of the OD matrix for a signalised junction is proposed. It proceeds in two steps :
 - An HLPN model provides a vehicle conservation law at each traffic light cycle
 - A fuzzy linear programming approach provides a point estimation endowed with a confidence interval
- Data, used to estimate the OD flow rates, are collected at an experimental junction
- The best results, obtained for $\alpha = 3$, are accurate and not significantly different from those obtained in our previous work (Biletska *et al.*). However, the confidence interval ensures a set of plausible solutions.

Future lines of research

- Modelling of the temporal uncertainty of the HLPN model using the notion of fuzzy timestamps
- Extension of the proposed method to a sequence of junctions

References I



[Midenet et al., 2004] Midenet, S., Boillot, F. and Pierrelée, J.-C. (2004)

Signalized intersection with realtime adaptive control : on-field assessment of CO₂ and pollutant emission reduction

Transportation Research Part D, 9, 29-47



[Jensen et al., 1998] Jensen, K., Kristensen, L. M. and Wells, L. (1998)

Coloured Petri nets and CPN tools for modelling and validation of concurrent systems

International Journal on Software Tools for Technology Transfer (Springer Verlag), 98-132



[Biletska et al., 2009] Biletska, K., Midenet, S., Masson, M.-H. and Denoeux, T. (2009)

Fuzzy modelling of the sensor data for the estimation of an origin-destination matrix.

To appear in Proceedings of the IFSA-EUSFLAT'09, July 20-24, Lisbon, Portugal.



[Murata 1989] Murata, T. (1989)

Petri nets : properties, analysis and applications

In Proceedings of the IEEE, Vol. 77, 541-580



[Dubois and Prade, 1980] Dubois, D. and Prade, H. (1980)

Fuzzy sets and Systems : Theory and Applications

Academic press, Inc.

Thank you for your attention !

