Stochastic Shortest Path Problem with Coordinated Traffic Signals

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Traffic signals on urban arterial

Source: Wu et al. 2011
Traffic signal applications

Green Driver (apple app store)

SMART-Signal system
http://dotapp7.dot.state.mn.us/smartsignal/
Shortest path problem with traffic signals

  - Static link travel time.
  - Fixed timing control.
- Ahuja et al. (2002).
  - Dynamic link travel time.
  - Fixed timing control.
  - Stochastic link travel time.
  - Adaptive control.
Markov decision process (MDP)

- States: $S$.
- Actions: $A_i, i \in S$.
- Costs: $c(i, a(i)), i \in S, a(i) \in A_i$.
- Transition probabilities: $P(j|i, a(i)), j \in S, i \in S$.
- Goal: minimize expected total cost.
State space

\[ S_v = \{(v, t) : v \in V_n, n \in N, t \in \{t_1, t_2, ..., t_{Y(v)}\}\} \]

- \(N\): set of intersections in the traffic network.
- \(V_n\): set of arrival nodes extended from \(n\).
- \(t\): possible arrival times.
Action and cost

- Action: \( a(i) \in A_i \subset V_N \) is a downstream node of \( v \).
- Cost: \( E(\delta_r + \delta_q + \tau|i, a) \)
  - \( \delta_r \): red light delay; \( \delta_q \): queuing delay; \( \tau \): link travel time.
Transition probability

\[ P(j|i, a) = P(t' = (t + \delta_r + \delta_q + \tau - \xi) \% \gamma | i, a) \]

- \( \delta_r \): red light delay; \( \delta_q \): queuing delay; \( \tau \): link travel time.
- \( \xi \) is the offset between these two coordinated intersections.
- \( \gamma \) is the cycle length at \( w \). \% is the modulo operation.
Objective function

Minimize

\[ u_\pi(s) = \lim_{X \to \infty} E\left\{ \sum_{k=0}^{X-1} c(i_k, a_k) | i_0 = s \right\}. \]
Cost calculation

Delays, Sep-09-2009

Delays, Sep-10-2009

Stochastic Shortest Path Problem with Coordinated Traffic Signals
Intersection delay histogram

Distr, Sep-09-2009

Distr, Sep-10-2009

Stochastic Shortest Path Problem with Coordinated Traffic Signals
Value iteration algorithm

Data: traffic network, signal settings, intersection delay distributions, link travel time distributions, destination
Result: expected time cost at each state, the navigation policy to the destination with minimal expected time cost

for each state $i \notin$ goal state do
  $u_0(i) = \infty$;
end

for each state $i \in$ goal state do
  $u_0(i) = 0$;
end

while $\forall i, \| u_k(i) - u_{k-1}(i) \| < \varepsilon$ do
  for each state $i$ do
    for each action $a$ do
      compute $Q_k(i, a) = c(i, a) + \sum_j P(j|i, a)u_{k-1}(j)$;
    end
    compute and store $\pi_k^*(i) = \arg\min_a Q_k(i, a)$;
    compute and store $u_k(i) = Q_k(i, \pi_k^*(i))$;
  end
return $\pi_k^*(i), u_k(i)$
Preliminary results – setting

- Cycle length: 4 unit time.
- Offset: 2 unit time.

<table>
<thead>
<tr>
<th>delay (unit time)</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>probability</td>
<td>0.1</td>
<td>0.8</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Link travel time distribution

<table>
<thead>
<tr>
<th>cycle time</th>
<th>delay</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>right</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>left</td>
<td>2.1</td>
<td>2.1</td>
</tr>
<tr>
<td>straight</td>
<td>0.1, 2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Intersection delay distributions
Preliminary results - test 1

<table>
<thead>
<tr>
<th>route</th>
<th>Time 1</th>
<th>Time 2</th>
<th>Time 3</th>
<th>Time 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2-3-4-5-6-12</td>
<td>14.758</td>
<td>13.989</td>
<td>13.250</td>
<td>12.933</td>
</tr>
<tr>
<td>1-7-8-9-10-11-12</td>
<td>15.375</td>
<td>15.075</td>
<td>14.575</td>
<td>14.375</td>
</tr>
</tbody>
</table>
# Preliminary results - test 1

<table>
<thead>
<tr>
<th>Offset = 2</th>
<th>Expected path time cost</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>route</td>
<td>Time 1</td>
<td>Time 2</td>
<td>Time 3</td>
<td>Time 4</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Offset = 1</th>
<th>Expected path time cost</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>route</td>
<td>Time 1</td>
<td>Time 2</td>
<td>Time 3</td>
<td>Time 4</td>
<td></td>
</tr>
<tr>
<td>1-2-3-4-5-6-12</td>
<td>15.018</td>
<td>14.787</td>
<td>14.410</td>
<td>14.504</td>
<td></td>
</tr>
<tr>
<td>1-7-8-9-10-11-12</td>
<td>15.375</td>
<td>15.075</td>
<td>14.575</td>
<td>14.375</td>
<td></td>
</tr>
</tbody>
</table>
**Preliminary results - test 2**

<table>
<thead>
<tr>
<th>Expected path time cost</th>
<th>route</th>
<th>Time 1</th>
<th>Time 2</th>
<th>Time 3</th>
<th>Time 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2-3-4-5-6</td>
<td>12.540</td>
<td>11.775</td>
<td>11.038</td>
<td>10.719</td>
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<tr>
<td>Node 7 to node 6</td>
<td>12.885</td>
<td>12.885</td>
<td>12.481</td>
<td>12.281</td>
<td></td>
</tr>
</tbody>
</table>
Future researches

• State space reduction
• Other solution methods
• Constrained shortest path problem
References 1


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References 2

Thank you!

Questions or suggestions?